Summary of Methods of Determining Convergence and Divergence

You need to be keeping a card or paper that you can refer to as we work our way though this chapter, with all the methods of finding out if a series is convergent or divergent. You will need to know them all for the test; you cannot use notes on the test. So far, we have:

1. **Series** are the sums of an infinite number of terms. They are a sequence of partial sums, and in order for the series to converge, the sequence of partial sums must converge to a finite number. You can test for convergence/divergence in a number of ways:

a) If you have a **geometric series**, the series converges if $|r| < 1$ and diverges if it does not.

b) You can use the **nth term test**. If the limit of the a_n term is <u>not</u> zero, then the series diverges. The fact that the limit of a_n is zero does <u>not</u> prove that the series converges; it may or may not.

c) If you have a **telescoping series**, you can write out some terms (usually using partial fractions) and find the sum. If there is a finite sum then the series converges.

d) In the **Ratio Test** if the limit as n approaches infinity of the absolute value of the ratio of terms a_{n+1} to a_n is less than one the series converges, if the limit is greater than one or infinite the series diverges, and if the limit equals one the test is inconclusive (which means you have to go to a different test).

e) In the **Root Test** if the limit as n approaches infinity of the n^{th} root of the absolute value of the term a_n is less than one the series converges, if the limit is greater than one or infinite the series diverges, and if the limit equals one the test is inconclusive.

f) You can use the **integral test**. If the expression for a_n is positive, continuous, and decreasing for all $x \geq 1$ then the series and the integral of the expression for a_n either both converge or both diverge.

g) If you have a **p-series**, $\sum_{n=1}^{\infty} \frac{1}{n^p}$, the series converges when $p > 1$ and diverge when $p \leq 1$.

h) The **Direct Comparison Test** lets you compare a positive termed series to one whose convergence is known. If your series is less than a known convergent series, then it will also converge. If it is greater than a known divergent series, then it will also diverge.

i) In the **Limit Comparison Test**, if the limit of the ratio of your series to a known series is a finite value, then both series either diverge or converge.

j) The **Alternating Series Theorem** says that if the terms in an alternating series approach zero as n approaches infinity and each term is less than the previous one, then the series will converge.